Teaching through Activity

Presentations by:
Matt Kaufmann – mlkaufm@ilstu.edu
Darl Rassi – mr.rassi@gmail.com

"Activities that are so cheap... yet so rich!"

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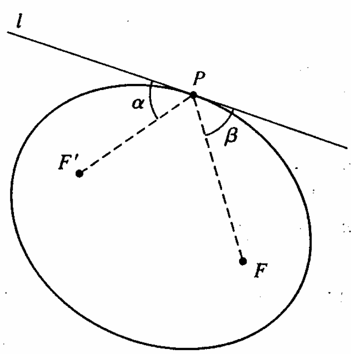
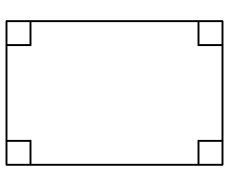
Figure 12



An ellipse has a ***reflective property*** analogous to that of the parabola discussed at the end of the previous section. To illustrate, let l denote the tangent line at a point P on an ellipse with foci F and F', as shown in Figure 12. If α is the acute angle between $F'P$ and l and if β is the acute angle between FP and l, it can be shown that $\alpha = \beta$. Thus, if a ray of light or sound emanates from one focus, it is reflected to the other focus. This property is used in the design of certain types of optical equipment.

If the ellipse with center O and foci F' and F on the x-axis is revolved about the x-axis, as illustrated in Figure 13, we obtain a three-dimensional surface called an **ellipsoid**. The upper half or lower half is a **hemi-ellipsoid**, as is the right half or left half. Sound waves or other impulses that are emitted from the focus F' will be reflected off the ellipsoid into the focus F. This property is used in the design of *whispering galleries*—structures with ellipsoidal ceilings, in which a person who whispers at one focus can be heard at the other focus. Examples of whispering galleries may be found in the Rotunda of the Capitol Building in Washington, D.C., and in the Mormon Tabernacle in Salt Lake City.



2. Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

Problem Solving with Skittles

BACKGROUND:

Students appreciate an open-ended challenge. Through this challenge the idea of algebraic substitution takes on meaning, and helps students be successful at solving systems of linear equations using the substitution method.

OBJECTIVES:

1. Students will gain an intuitive and explicit understanding of algebraic substitution.
2. Students will learn the importance of systematic guessing and checking.
3. Subscripted variables are introduced in a way that makes sense to the students.

GRADE LEVEL:

Pre-Algebra, Algebra 1, Algebra 2

CLASS TIME:

45 minutes

LESSON PLAN:

- Presentation of the Problem (5 min.) – Make sure that students understand the problem.
    - There are 5 paper bags.
    - There are 100 Skittles total in all the bags.
    - The first and second have 52 Skittles.
    - The second and third have 43 Skittles.
    - The third and fourth have 34 Skittles.
    - The fourth and fifth have 30 Skittles.
    - How many Skittles are in each bag?
- Work Time (15-20 min.) – Give the students 20-30 minutes to work in groups or with a partner.
    - Help students to stay on task. If they have no idea how to start lead them to the guess and check method. Say, "What if there are 10 Skittles in the first bag? Would that help? Is that correct?"
    - After a student or students find the solution, they should not shout it out. Ask them to find the solution, if there are 110 Skittles total instead of 100.
    - If students find this solution, ask them "What is the largest amount of Skittles that could be in these bags if the 4 statements are true? What is the smallest amount?"
- Presentation (10-15 min.) – After most all students have found the solution for the first question, have them present their methods for finding the solution.
    - Have students present as many different ways of finding the solution as possible.
    - Make sure to point out the importance of systematic guessing and checking as opposed to random guessing and checking.
    - If no one has presented the "quick" solution, do so.
- "Quick" Solution –

- We know that $Bag\,1 + Bag\,2 + Bag\,3 + Bag\,4 + Bag\,5 = 100$. (Here I introduce subscripted variables and say does it make sense if I write $B_1 + B_2 + B_3 + B_4 + B_5 = 100$?)
- "We know how many are in Bags 1 and 2, right?" So instead of saying $B_1 + B_2 + B_3 + B_4 + B_5 = 100$ we can say $52 + B_3 + B_4 + B_5 = 100$.
- Then, I really focus on why we can do this. I give the example of 5 $1 bills and 1 $5 bill. "Are they equal to the same amount?" "If I trade them will I still have the same amount?" "Are they the same thing?" "Can I substitute one in place of the other?"
- Then I ask, "Bags 1 and Bags 2 equal to 52? Are they the same thing? Can I substitute one in for the other?"
- Then, I say, "Here can I replace $B_1 + B_2$ with 52? Why?"
- From here, you can ask if anyone sees how to find the answer quickly, or you can just show them that $B_3 + B_4 = 34$ and that another substitution makes it easy to find B_5. Once you know B_5 it is easy to find everything else.

- Extension to a system of linear equations (5-10 min.)
  - Now, I tell them that all of the problems we do won't involve Skittles and Bags, but the same ideas apply. Now we are going to do it with variables.
  - I give the students two equations like $y = 2x$ and $x + y = -12$. I then ask them if they can find x and y.
  - There are several ways to approach this. You can do it in class or you can have them work on it with a partner or individual.
  - The important part is in the explanation, whether it is by the teacher or another student. When students say to put 2x in for y, ask the same questions that were asked earlier about the bag problem. "Why can we do that? Are they the same thing? Can I substitute one in for the other?"
  - This is a fundamental idea for solving a system of equations that most students do not understand intuitively. This activity gives students an understanding of what it means to substitute and why it is okay to substitute.

STANDARDS:

- Algebra Standard: Represent and analyze mathematical situations and structures using algebraic symbols.
- Problem Solving Standards
- Communication Standards
- Representation Standards

MATERIALS:

- 5 Brown Paper Bags
- 100 Skittles (I've found that the bags could actually be empty).
- One overhead with the Problem Statement

Graphing Stories (an introduction to graphing)

OBJECTIVES: 1. Students should understand that every graph tells a specific story. This story is articulated by the labeling, units and plotted coordinate points.
2. Students should be able to interpret such a story from a graph and draw a graph relating to a story.

OPENER (10 min): Using a CBR and an overhead projection screen the instructor will generate a distance vs. time graph by walking back and forth in front of a CBR for the whole class to see. The class will be asked questions to elicit understanding about the meaning of a graph.

Questions to prompt student thinking:

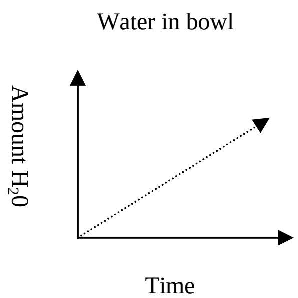
1. What story is being told by the graph?
2. Who/what is generating the graph?
3. What information does the graph tell?
4. What markings on the graph help us understand its meaning?

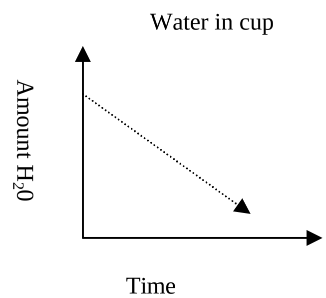
(The teacher should run the demonstration as many times as necessary making his/her movements obvious so that students see the relationship between the teacher's movement and the generated graph.)

TASK 1 (15 min): Students working in pairs will use TI-83 calculators and CBR's and run the Dist-Match program.

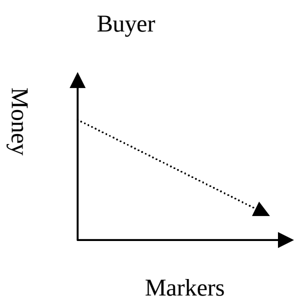
TASK 2: Students will work in groups of 2 to draw various graphs on marker boards that "tell the story" in a demonstration acted out by the teacher. There are many correct answers. Each graph should tell the story without the aid of an explanation.

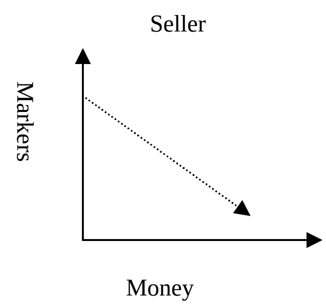
1st demonstration – A cup of water is poured into a bowl.





2nd demonstration - A simulation of a buyer and a seller exchanging money and goods (markers) will be demonstrated.





Assessment: Students graph "The Tortoise and the Hare" story.

Conic Creations

ABSTRACT:
It is important for students to see mathematics from start to finish. All too often, the application of math is distant and unfathomable. In this two day activity, students use their knowledge of mathematics to build solar collectors using parabolas, wave and reflective pools using the ellipse and hyperbolic reflectors using hyperbolas. At the end of the activity, students will observe the application of mathematics in the work of their hands.

LEVEL: Algebra 2, Pre-Calculus

OBJECTIVES:
- Use mathematics to build conic models
- Observe optic properties

TASK 1:
Groups of students will design and build one specific conic model

1. parabolic solar collector
2. elliptic wave pool
3. elliptic light reflector
4. hyperbolic light reflector

TASK 2:
Each team of students will observe and document the optical properties of their model.

TASK 3:
Each team will present their conic model and its optic properties to the rest of the class.

TIME NEEDED: one day for task 1, one day for task 2 & 3

STANDARDS:
9.B.5 Construct and use two- and three-dimensional models of objects that have practical applications
8.B.4a Represent algebraic concepts with physical materials, words, diagrams, tables, graphs, equations

FILES IN ZIP
- Solar collector worksheets
- Ellipse wave pool worksheets
- Ellipse light reflector worksheets
- Hyperbola light reflector worksheets

Solar Collectors

Design/Build

MATERIALS: 1 piece of styrofoam, 1 sheet of graph paper, 1 graphing calculator, 1 copper tube, 1 strip of cardstock, 1 strip of tin foil, 10 thumb tacks, 1 utility knife, 1 marker, 1 scissors, scotch tape

1) Use the graphing calculator to find the equation of a wide parabola.

equation: ___________________

2) Sketch the parabola on the graph paper. Be as accurate as possible!

3) Mark where the focus of the parabola is on your graph paper.

4) Using a scissors, cut out the parabola.

5) Use your parabola cut out as a pattern. Using a marker, trace two parabolas on the Styrofoam and mark the focus on each parabola.

6) Using a utility knife, carefully cut the parabolas out of the Styrofoam.

(Hint: Score the Styrofoam. Break off the waste pieces.)

7) Push a pencil through the focus of each Styrofoam parabola to create a hole. Next, link the two parabolas together by pushing the copper tube through the focus of each parabola.

8) Wrap card stock around both curves of the Styrofoam parabolas fastening it on using thumb tacks.

9) Smooth out the tin foil strip with your hands. Line the inside of the solar collector with the tin foil and fasten it with scotch tape.

Solar Activity Worksheet

$$Q = mc?T$$

Q = heat, m = mass, c = specific heat,
? T = change in temperature

MATERIALS: Solar collector, 2 Thermometers

Insert the temperature probe into the copper piping. Set the collector in the sun and measure the heat absorbed by collector.

1) Mass of the copper tube: _________________

2) Specific heat of copper: ______________

3) Initial temperature: _______________

4) Final Temperature: _______________

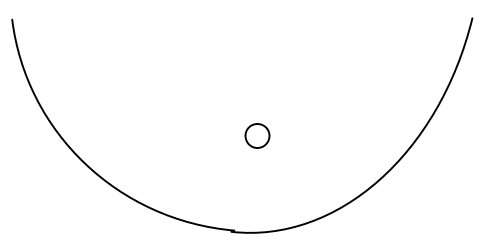
5) Change in Temperature: ______________

6) Time in sun: ________________

7) Calculate the heat absorbed in your solar collector.

8) Calculate the heat absorbed per square inch of collectable solar area.

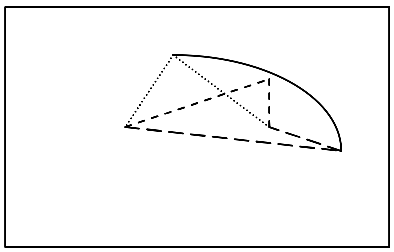
9) On the figure below draw the light ray reflections.



Elliptic Laser Show
Design/Build

MATERIALS: 1 piece of styrofoam (16 X 24), 1 piece of string (12 – 18 in.), 15 thumb tacks, 1 marker, 1 utility knife, strips of laminated paper, strips of tin foil, scotch tape

1) Insert two thumb tacks on a piece of styrofoam approximately 6-12 inches apart. Tie each end of the string to one of the tacks. Scribe the curve that is bounded by the string when it is pulled tight. (view the figure below)



2) What type of conic section is drawn?

3) Write the equation that closely matches the sketched curve (use a ruler and round measurements to the nearest inch).

4) Score the curve with a utility knife. Run the blade over the incision a few times to cut the groove deeper. Carefully break off unwanted pieces leaving the ellipse intact.

5) Use thumb tacks to pin the laminated paper walls to the outside of the ellipse.

6) Smooth out the tin foil with your hand and fasten it on to the interior walls with scotch tape (try to get a smooth and tight fit).

Elliptic Laser Show
Activity Worksheet

MATERIALS: Ellipse reflecting pool, 1 laser pointer, 1 cylinder block (1x3in.)

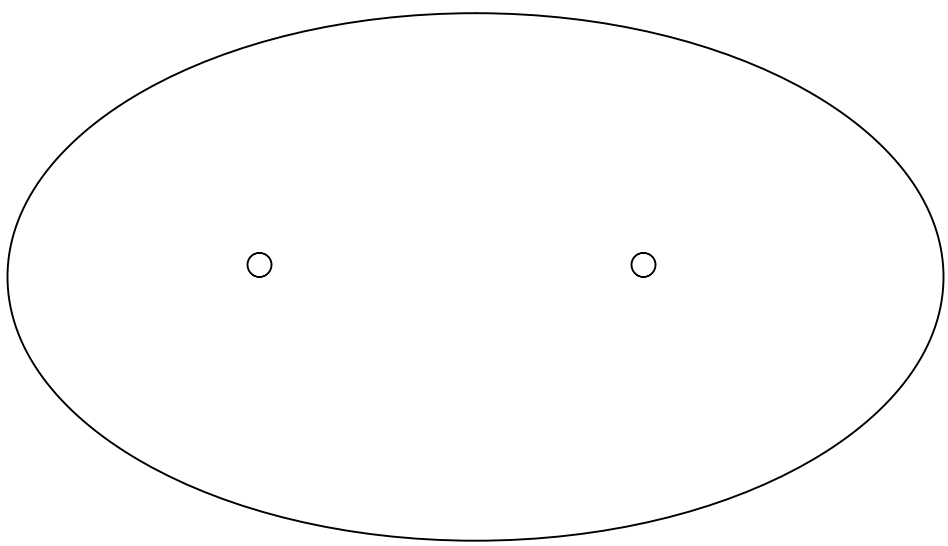
1) Use a marker to mark both of the foci inside the ellipse (they are the tack holes used to construct the ellipse).

2) Place the cylinder block on top of one focus point.

3) Shine the laser parallel with the surface of the ellipse on top of the other focus point. Slowly turn the laser 360 degrees.

4) What observations can you make?

5) On the figure below, draw what is happening.



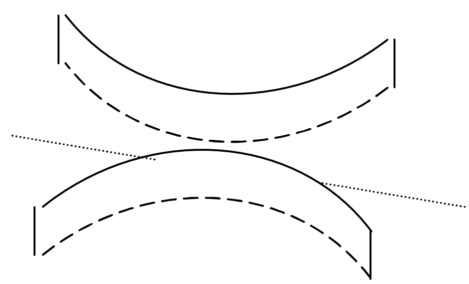
Hyperbolic Light Reflector
Design/Build

MATERIALS: 1 piece of styrofoam (10X10in) with graph paper overlaid, strips of cardstock, strips of tin foil, scotch tape, 1 marker, 1 utility knife

1) Sketch $y^2/_9 - x^2/_{16} = 1$ on the graph paper (include asymptotes and foci).

2) Score both of the curves (branches only!) with a utility knife. Run the blade over the incision a few times to cut the groove deeper.

3) Push the card stock strips into both incisions.



4) Smooth out the tin foil with your hand. Using scotch tape, fasten it on to the sides of the walls that face each other (try to get a smooth and tight fit).

Hyperbolic Light Reflector
Activity Worksheet

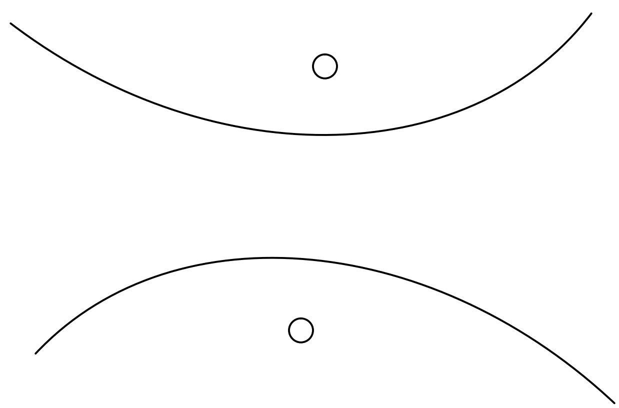
MATERIALS: Hyperbolic model, 1 laser pointer

1) Point the laser at one of the focus points but allow the beam to be intercepted by the tin foiled surface of the hyperbolic wall.

2) What observations can you make about the reflection of the laser beam?

3) Confirm your conjecture by trying part 1 again from a different starting point.

4) On the figure below, draw what is happening.



Standard Deviation
(using pop corn)

ABSTRACT: Introduce the concept of statistical variance while your students munch on pop corn. This activity can serve as a springboard into discussions regarding measures of central tendency, displaying data, variance and inference. This activity stimulates all five senses!!

TIME: 1 Class period

AGE: 7^{th} – 12^{th}

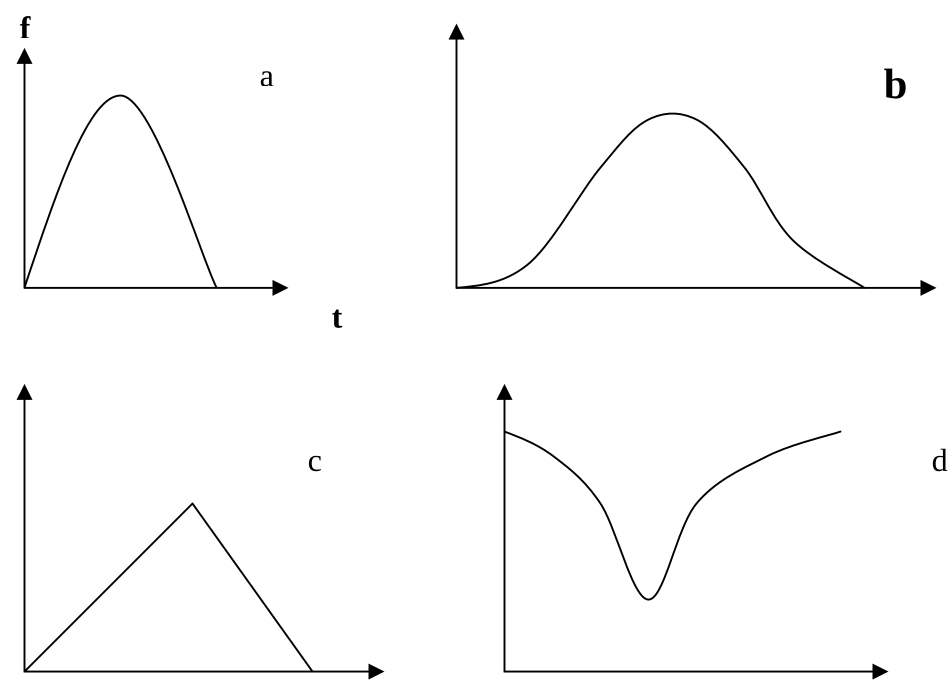
OBJECTIVES:

1. Introduce the Bell Curve.

2. Introduce variance

3. Introduce Standard Deviation

TASK 1: Pop popcorn in a "stir crazy." Students should choose the graph that best describes frequency of pops vs. time (as they listen to the pop-corn pop)



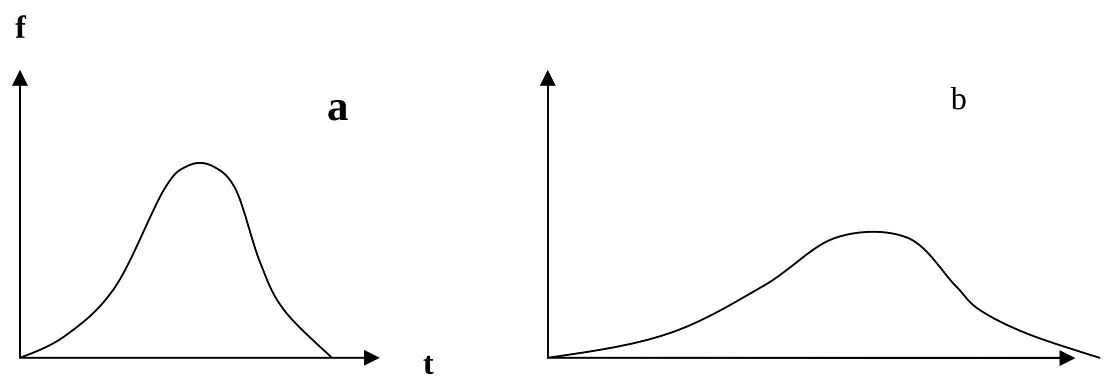
TASK 2: Students dip out "dixie cup" scoops of pop corn and count the kernels. We make a histogram of this data and talk about mean, median, mode, range and standard deviation. Students are instructed on how to calculate standard deviation on the graphing calc.

TASK 3: Students are led in a discussion about standard deviation.

* Standard deviation tells us on average how far each data point lies from the mean. Large S.D. indicates wide spread. Small S.D. indicates narrow spread.

* Question: Suppose I sell popcorn at a movie theater. Based on our sales and inventory I know the night Terri worked she filled popcorn bags with an average of 20.5 grams of popcorn with a S.D. of 2 grams. When Steve worked he bagged an average of 20.9 grams of popcorn with a S.D. of 5 grams. What does this information tell me? (**Terri scoops are consistent while Steve has more variance in his scoops**)

TASK 4: If I was a pop-corn producer, would I want a high or low S.D. for pop times? (**low**) Why (**think about microwave popcorn and the fear of burning it**)? Which graph represents this?



TASK 5 (Extension): Suppose a hybrid researcher was trying to improve the standard deviation of a strain of pop corn. If he takes a sample of his new corn and it pops with an improved S.D. can he conclude that he has made an improvement to the strain? What kind of tests could the researcher do to prove or disprove improvement?

***Think about sample sizes, repeating samples, S.D. of S.D., Empirical Rule, inference...**

STANDARDS:

Data Analysis: Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.

Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

The Boat Challenge

ABSTRACT: One of the most powerful tools mathematics offers industry is optimization. The ability to predict and design the best "specimen" given various goals and constraints is lucrative. The "Boat Building Challenge" places students in a situation in which the mathematically savvy and construction apt student will shine. Student teams are given one laminated 8.5 x 11 piece of paper. They are asked to build the boat that will hold the most weight. (i.e. the boat with the greatest amount of volume)

Implementation

The *Boat Building Challenge* can be implemented in many different ways. Variations of two methods are commonly used.

Method 1. Teacher Directed

Through deliberately planned tasks, the teacher initiates discovery.

Time Requirement: Requires 3 class periods, and some out of class student work.
Example: Refer to the detailed lesson plans below.

Method 2. Student Directed

Motivated by the challenge, students invent and discover solutions on their own.

Time Requirement: Requires 15 minutes to initiate the challenge, 25 minutes to test boats periodically.

To view solutions for some boat designs go to www.gk12.ilstu.edu/presentation/solutions

User Name: teacher

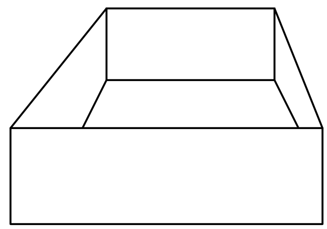
Password: project

Example of 3 day lesson (method 1):
(A power point instructional tool is provided for this 3 day series)

DAY 1: (50 minute period)

Give students an 8.5 x 11 piece of laminated paper, roll of scotch tape, scissors, ruler

A. Students are asked to build a rectangular boat that will hold the most weight before sinking in a basin of water. (20 min)



B. Test boats in buckets of water. (10 min) (*pg.1)

C. Observe the best and worst boats. The best will have the most volume the worst will have the least volume. Buoyancy: a boat floats when it displaces its weight in water. Therefore we want the boat with the most volume. (*pg.2,3)

D. Show optimal boat by using mathematics. (20 min)

$f(x)$ = volume = (length)(width)(height) = $(11 - 2x)(8.5 - 2x)(x)$

The function is maximal at $x \sim 1.585$ inches, $f(1.585) \sim 66.15$ in^3

(NOTE: Waste has been discarded) (*pg.4,5)

E. Observe that the optimal boat floats the most weight. (*pg.6)

F. Issue a new challenge to individuals as homework. Design the best “canoe” – one fold down the middle and ends are parallel. Students should bring to class a physical model. (*pg.7)

*Refers to page number in the student workbook.

DAY 2: (50 minute period)

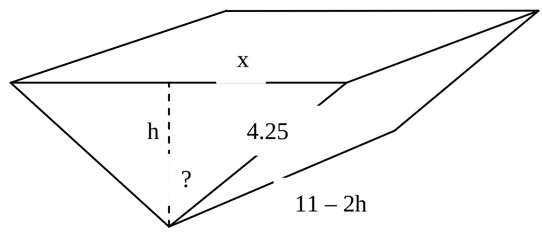
Students come to class with their already built canoe boats.

A. Discuss variables/parameters/constraints. In groups, write a function for volume. (25 min)

i. $f(h) = h(18-h^2)^{0.5}(11-2h)$

ii. $g(x) = x(18-x^2)^{0.5}(11-2(18-x^2))$

iii. k(?) = 4.25^2cos?sin?(11-8.5cos?) (*pg.8,9)



B. Students should check the accuracy of their functions by graphing them on a graphing calculator and analyzing the graph to determine if their function is reasonable. (i.e. do the x and y intercepts, asymptotes, domain, range make sense?) (10 min) (*pg.10)

C. Have students put up correct derivations (functions f(h), g(x), k(?)) on marker board. (15 min) (*pg.11)

D. Graph f(h), g(x) and k(?) and optimize the function and interpret your answer. (homework)

i. h = 2.16 inches, f(2.16) = 52.69 inches cubed

ii. x = 3.65 inches, g(3.65) = 52.69 inches cubed (*pg.10)

iii. ? = 59.37 degrees, k(59.37) = 52.81 inches cubed (*pg.12)

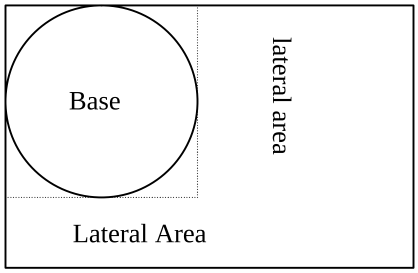
E. Issue a new challenge to the groups. Give groups a new laminated 8.5 x 11 sheet of paper. Design the "best" boat to be tested in a week. Any design!!! (*pg.13)

DAY 3: (50 minute period)

A. Test Boats

B. Students explain their designs and the mathematics for optimization.

C. If time is needed students should (if haven't already) investigate the cylinder.



Discuss the optimal volume from three perspectives:

1st When waste is ignored
2nd When the waste is used to extend height
3rd When the waste is included in the function to extend all the dimensions

1) Volume function when waste is ignored:

f(r) = (area base)(height)
= pr^2 [(11·8.5) – (2r)(2r)] / 2pr
= $-2r^3 + 46.75r$ Optimal: radius ~ 2.79 inches; volume ~ 87 in^3

2) Volume when the previous waste is used to extend height
6.7 in^2 of waste material is used to gain 0.382 inches to height giving a volume of 96.3 in^3

3) Volume function when waste is calculated into the function.

f(r) = p r^2 [(11·8.5) – (p r^2)] / 2 p r
= - $^p/_2$ r^3 + 46.75r Optimal: radius ~ 3.15 inches; volume ~ 98.2 in^3

CONCLUSION: Using waste pays off. Using waste wisely pays off even better. Incorporating the waste in the design function optimizes the use of your waste.